Pricing Energy Storage for Social-Welfare Maximization

Ning Qi, Ningkun Zheng, Bolun Xu

nq2176@columbia.edu

Columbia University, New York, USA

October 22, 2024

Bio

Name: Ning Qi **Title: Postdoctoral Research Scientist**

Affiliation: Earth & Environmental Engineering, Columbia University

Email: nq2176@columbia.edu **Homepage: https://thuqining.github.io**

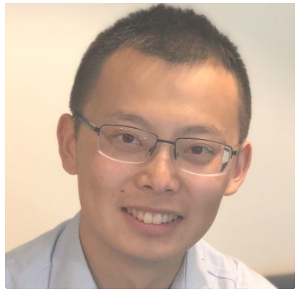
Research Interest: Data-Driven Modeling, Optimization under Uncertainty and Market Design for Power System with Generalized Energy Storage Resources

- B.S. EE, Tianjin University, 2018 — Prof. Yanxia Zhang
- PhD. EE, Tsinghua University, 2023 — Prof. Lin Cheng
- Visiting Scholar, Technical University of Denmark, 2022
- Imperial College London — Prof. Pierre Pinson
- University of Vermont — Prof. Mads R. Almassalkhi
- Research Associate, Tsinghua University, 2023 — Prof. Feng Liu
- Postdoc, Columbia University 2024 — Prof. Bolun Xu

Group Member:

Ningkun (Nik) Zheng

Neal Ma



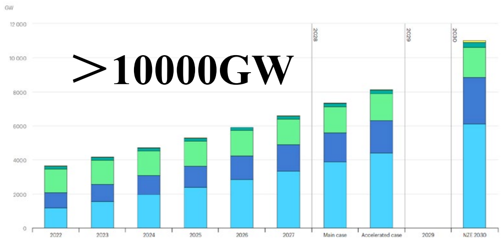
Bolun Xu

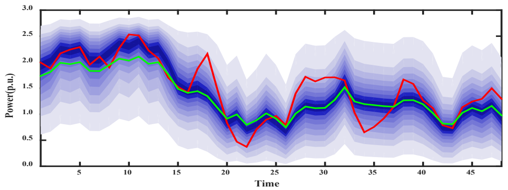
Group Homepage:

https://bolunxu.github.io/

Background—Role of Energy Storage

Renewables

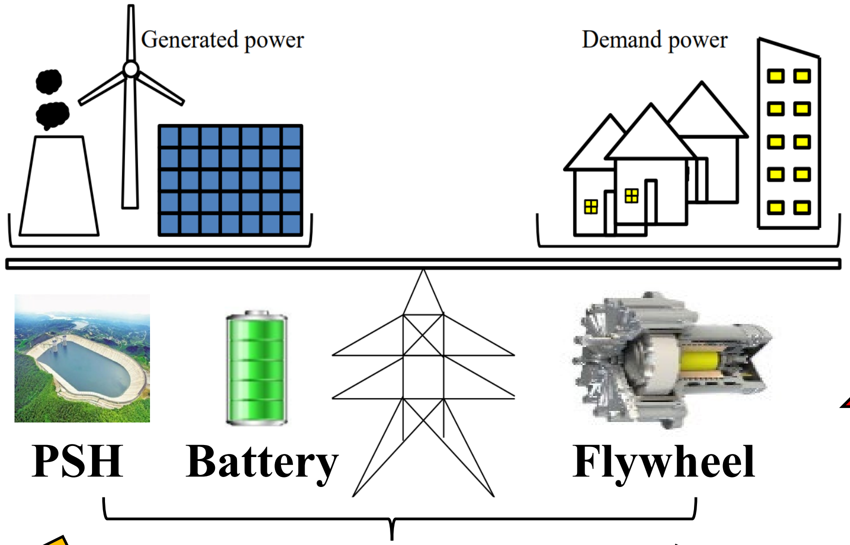




Capacity ↑

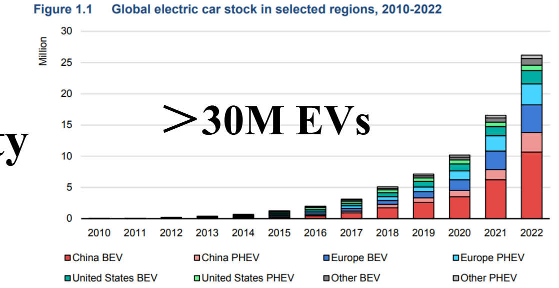
Uncertainty ↑

Power Supply-Demand Balance



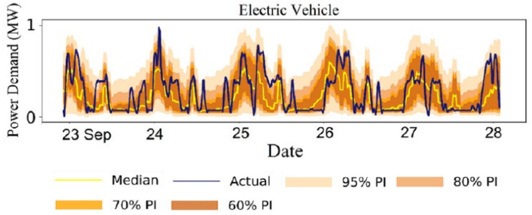
Energy Storage

Flexible Loads



Capacity ↑

Uncertainty ↑

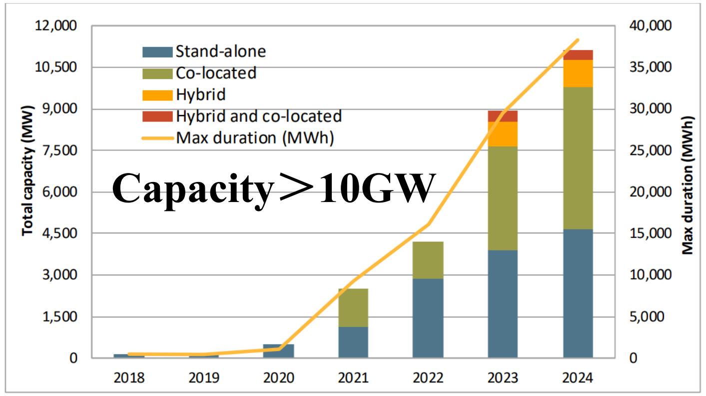


Multiple Services

- Energy Shifting (Price Arbitrage)
- Frequency Regulation
- Capacity Market…

?

Pricing



Background—Pricing of Energy Storage

☺

Cost Recovery

Conventional Generator

Market Design

VS

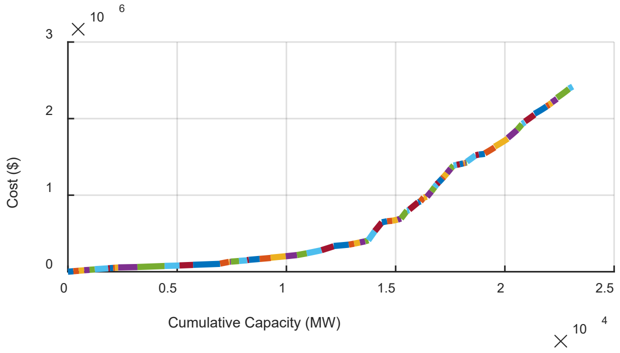
Energy Storage

☹

Small Storage

Lose Money
Without Intelligence

	Conventional Generator	Energy Storage
Bid:	Marginal Cost	Opportunity Cost
Bid Generation:	Fuel Cost Curve	Price Prediction based on Private Model
Market Clearing:	Marginal Energy Price	No Default Bid or Price



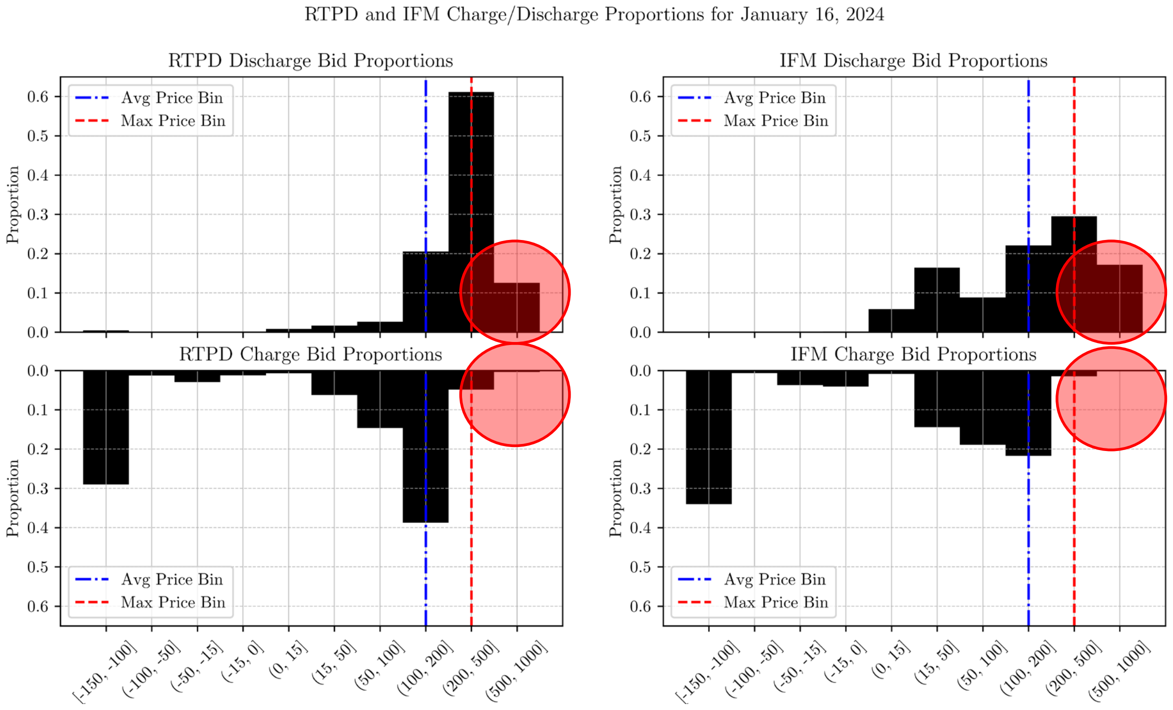
$$Q_{t-1}(e_{t-1}|\hat{\lambda}_t) := \max_{p_t, b_t} \hat{\lambda}_t (p_t - b_t) - c p_t + V_t(e_t)$$

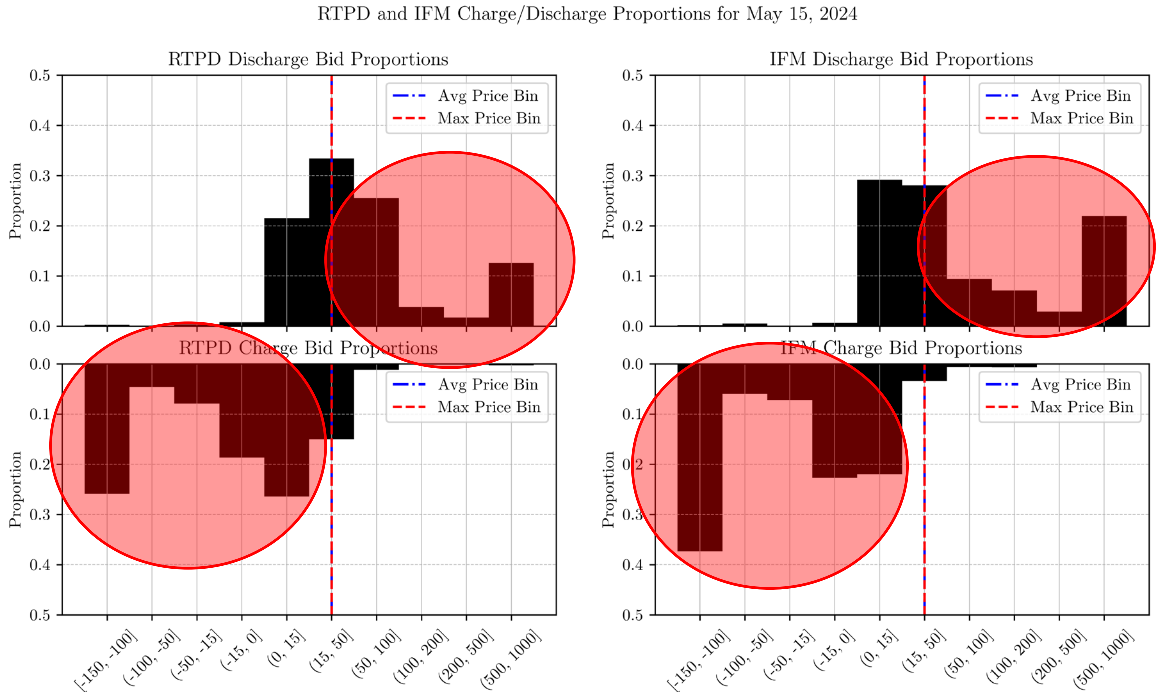
$$V_{t-1}(e_{t-1}) := \mathbb{E}_{\hat{\lambda}_t}\left[Q_{t-1}(e_{t-1}|\hat{\lambda}_t)\right]$$

Profit Maximization

Background—Pricing of Energy Storage

Large Storage: Capacity Withholding (Extreme High Bid: $500-1000/MWh!)





Normal Price Day **Default Bid?** **Price Spike Day**

https://www.caiso.com/library/daily-energy-storage-reports

Background—Pricing of Uncertainty

Deterministic Formulation

$$\min \sum_t [\sum_i G_i(g_{i,t}) + \sum_s M_s(p_{s,t})] \quad (1a)$$

$$[\lambda_t]: \sum_i g_{i,t} + \sum_s p_{s,t} - \sum_s b_{s,t} = D_t, \forall t \in \mathcal{T} \quad (1b)$$ **Energy Price**

$$[\theta_{s,t}]: e_{s,t+1} - e_{s,t} = -p_{s,t}/\eta_s + b_{s,t}\eta_s, \forall t \in \mathcal{T}, \forall s \in \mathcal{S} \quad (1c)$$

$$[\pi_t]: \sum_i \varphi_{i,t} + \sum_s \psi_{s,t} \geq d_t, \forall t \in \mathcal{T} \quad (1d)$$ **Reserve Price**

$$[\underline{\nu}_{i,t}, \overline{\nu}_{i,t}]: \underline{G}_i \leq g_{i,t} \leq \overline{G}_i - \varphi_{i,t}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G} \quad (1e)$$

$$[\underline{\kappa}_{i,t}, \overline{\kappa}_{i,t}]: -RD_i \leq g_{i,t} - g_{i,t-1} \leq RU_i, \forall t \in \mathcal{T}, \forall i \in \mathcal{G} \quad (1f)$$

$$[\underline{\alpha}_{s,t}, \overline{\alpha}_{s,t}]: 0 \leq b_{s,t} \leq \overline{P}_s, \forall t \in \mathcal{T}, \forall s \in \mathcal{S} \quad (1g)$$

$$[\underline{\beta}_{s,t}, \overline{\beta}_{s,t}]: 0 \leq p_{s,t} \leq \overline{P}_s - \psi_{s,t}, \forall t \in \mathcal{T}, \forall s \in \mathcal{S} \quad (1h)$$

$$[\underline{\iota}_{s,t}, \overline{\iota}_{s,t}]: (\psi_{s,t} + p_{s,t})/\eta_s + \underline{E}_s \leq e_{s,t}, \quad (1i)$$
$$e_{s,t} \leq \overline{E}_s - b_{s,t}\eta_s, \forall t \in \mathcal{T}, \forall s \in \mathcal{S}$$

- **Price and Dispatch Decoupled with Uncertainty**
- **Reserve Price≈0，Lose Profit for Reserve Provision**
- ✓ **Incorporate Uncertainty into Pricing and Dispatch**

Probabilistic Formulation

☐ Stochastic Optimization

× Revenue adequacy and cost recovery for each scenario
× Scenario-Based Price, No Default Price
✓ Market Analysis × Market Clearning

☐ Robust Optimization

✓ Conservative

☐ Chance-Constrained Optimization

✓ Tractable
✓ Scalable
✓ Control the Risk

$$\mathbb{P}\left(a_i(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\xi}(\boldsymbol{x}) \leq b_i(\boldsymbol{x})\right) \geq 1 - \epsilon$$

[1] J. Kazempour, P. Pinson, and B. F. Hobbs, "A stochastic market design with revenue adequacy and cost recovery by scenario: Benefits and costs," *IEEE Transactions on Power Systems*, vol. 33, no. 4, pp.3531–3545, 2018.
[2] Y. Dvorkin, "A chance-constrained stochastic electricity market," *IEEE Transactions on Power Systems*, vol. 35, no. 4, pp. 2993–3003, 2019.

Problem Formulation & Preliminary

Two-Stage Chance-Constrained Economic Dispatch

$$\min \mathbb{E}\sum_t [\sum_i G_i(g_{i,t}+\varphi_{i,t}\boldsymbol{d_t})+\sum_s M_s(p_{s,t}+\psi_{s,t}\boldsymbol{d_t})] \quad (1a)$$

Expectation

$$[\lambda_t]: \sum_i g_{i,t}+\sum_s p_{s,t}-\sum_s b_{s,t}=D_t,\ \forall t \quad (1b)$$

Energy Price

$$[\theta_{s,t}]: e_{s,t+1}-e_{s,t}=-p_{s,t}/\eta_s+b_{s,t}\eta_s,\ \forall t,\ \forall s \quad (1c)$$

Storage Price

$$[\pi_t]: \sum_i \varphi_{i,t}+\sum_s \psi_{s,t}=1,\ \forall t \quad (1d)$$

Reserve Cost

$$0\le \varphi_{i,t},\psi_{s,t}\le 1,\ \forall t,\ \forall i,\ \forall s \quad (1e)$$

$$[\underline{\nu}_{i,t},\ \overline{\nu}_{i,t}]: \mathbb{P}\big(\underline{G}_i\le g_{i,t}+\varphi_{i,t}\boldsymbol{d_t}\le \overline{G}_i\big)\ge 1-\epsilon,\ \forall t,\ \forall i \quad (1f)$$

$$[\underline{\kappa}_{i,t},\ \overline{\kappa}_{i,t}]: -RD_i\le g_{i,t}-g_{i,t-1}\le RU_i,\ \forall t,\ \forall i \quad (1g)$$

$$[\underline{\alpha}_{s,t},\ \overline{\alpha}_{s,t}]: 0\le b_{s,t},\ \mathbb{P}\big(b_{s,t}-\psi_{s,t}\boldsymbol{d_t}\le \overline{P}_s\big)\ge 1-\epsilon,\ \forall t,\ \forall s \quad (1h)$$

$$[\underline{\beta}_{s,t},\ \overline{\beta}_{s,t}]: 0\le p_{s,t},\ \mathbb{P}\big(p_{s,t}+\psi_{s,t}\boldsymbol{d_t}\le \overline{P}_s\big)\ge 1-\epsilon,\ \forall t,\ \forall s \quad (1i)$$

$$[\underline{\iota}_{s,t},\ \overline{\iota}_{s,t}]: \mathbb{P}\Big((\psi_{s,t}\boldsymbol{d_t}+p_t)/\eta_s\le e_{s,t} \le \overline{E}_s-(b_{s,t}-\psi_{s,t}\boldsymbol{d_t})\eta_s\Big)\ge 1-\epsilon,\ \forall t,\ \forall s \quad (1j)$$

Chance-Constraints

$$b_{s,t}p_{s,t}=0,\ \forall t,\ \forall s \quad (1k)$$

Complementary Constraints

- First-Stage: Pre-dispatch $g_{i,t}\ p_{s,t}, b_{s,t}, e_s, \varphi_{i,t}, \psi_{s,t}$
- Second-Stage: Re-dispatch $\varphi_{i,t}\boldsymbol{d_t}\ \ \psi_{s,t}\boldsymbol{d_t}$

✓ Opportunity Pricing for Storage

✓ Default Bid and Benchmark the Market Power of Storage

Deterministic Reformulation

$$\mathbb{E}\big(G_i(g_{i,t}+\varphi_{i,t}\boldsymbol{d_t})\big)=\sum_{j=0}^{n} C_j\sum_{k=0}^{j}\binom{j}{k} g_{i,t}^{j-k}\varphi_{i,t}^{k}\mathbb{E}(\boldsymbol{d}_t)^k$$

$$\mathbb{E}\big(M_s(p_{s,t}+\psi_{s,t}\boldsymbol{d_t})\big)=M_s(p_{s,t}+\psi_{s,t}\mu_t)$$

$$\mathbb{E}(\boldsymbol{d}_t)^k=\sum_{0\le j\le k}^{j\ even}\binom{k}{j}\mu_t^{k-j}\sigma_t^{j}(j-1)!!$$

$$\underline{G}_i\le g_{i,t}+\varphi_{i,t}\hat{d}_t,\ g_{i,t}+\varphi_{i,t}\tilde{d}_t\le \overline{G}_i$$

$$b_{s,t}-\psi_{s,t}\hat{d}_t\le \overline{P}_s,\ p_{s,t}+\psi_{s,t}\tilde{d}_t\le \overline{P}_s$$

$$(\psi_{s,t}\tilde{d}_t+p_{s,t})/\eta_s\le e_{s,t},\ e_{s,t}\le \overline{E}_s-(b_{s,t}-\psi_{s,t}\hat{d}_t)\eta_s$$

$$\hat{d}t=\mu_t-F^{-1}(1-\epsilon)\sigma_t,\ \tilde{d}t=\mu_t+F^{-1}(1-\epsilon)\sigma_t$$

- Solve MILP with Binary Variables
- Substitute Binary Variables with Solution
- Resolve LP

Robust Competitive Equilibrium !

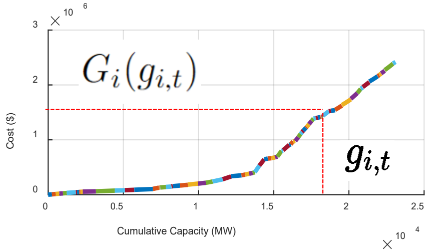
Theoretical Analysis

- **Proposition 1: Physical Cost Origins: (1) Marginal Cost of Cleared Generator & Storage (2) Storage Efficiency**

$$\theta_{s,t}=H_i(g_{i,t}+\varphi_{i,t}\boldsymbol{d_t})/\eta_s$$ **Charging**

$$\theta_{s,t}=\eta_s(H_i(g_{i,t}+\varphi_{i,t}\boldsymbol{d_t})-M_s)$$ **Discharging**

$$\partial\mathbb{E}G_i(g_{i,t}+\varphi_{,t}\boldsymbol{d_t})/\partial g_{,t}=H_i(g_{i,t}+\varphi_{i,t}\boldsymbol{d_t})$$

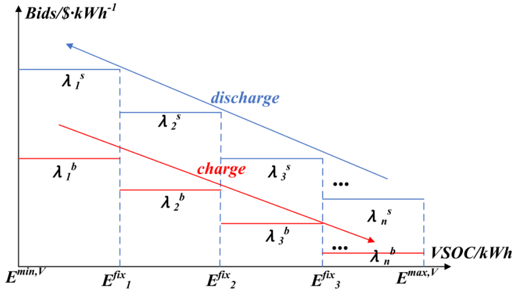


- **Proposition 2: Convex Opportunity Price: Monotonically Decreases with Storage SoC**

$$\partial\theta_{s,t}/\partial e_{s,t}\leq 0$$

Quadratic/Super-Quadratic Generation Cost Function

SoC-Dependent Bid[1]



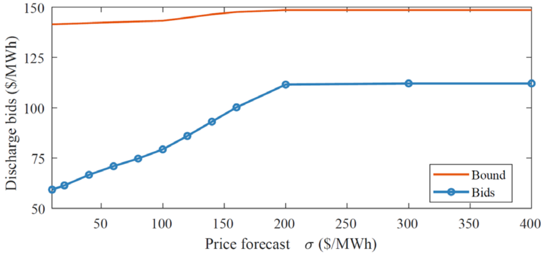
- **Proposition 3: Uncertainty-aware Price: Monotonically Increases with Uncertainty**

$$\partial\theta_{s,t}/\partial\sigma_t\geq 0$$

Quadratic/Super-Quadratic Generation Cost Function

$$\mathbb{E}(\theta_{s,t}(\boldsymbol{d_t}))>\theta_{s,t}(\mathbb{E}(\boldsymbol{d_t}))$$

Uncertainty-Aware and Bounded Bid[2]



- **Theorem: Constrained and Bounded Price**

$$\theta_{s,t-1}=\frac{\eta_s}{\widetilde{d}_t}(\theta_{s,t}\eta_s\widehat{d}_t+\lambda_t(\frac{\widetilde{d}_t}{\eta_s^2}-\widehat{d}_t)+\pi_t-M_s\mu_t)$$ **Charging**

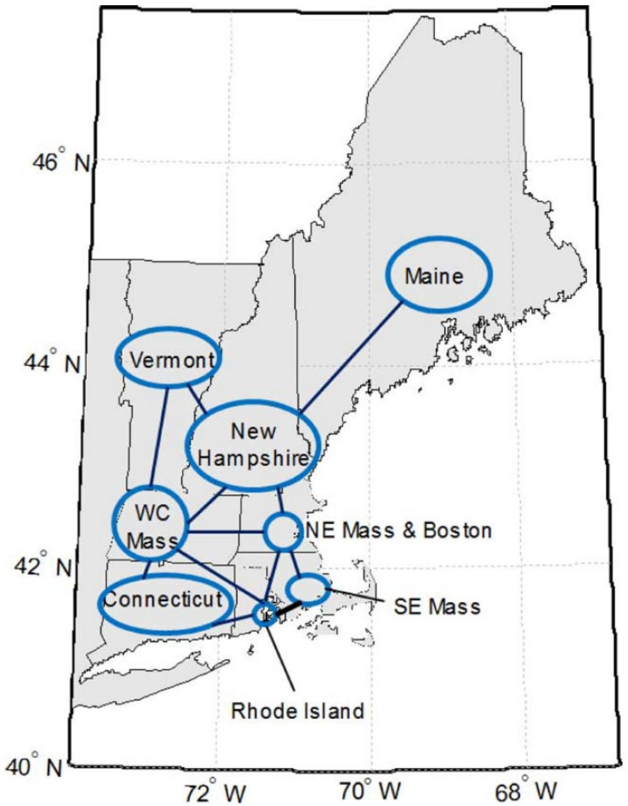
$$\theta_{s,t-1}=\frac{1}{\eta_s\widehat{d}_t}(\frac{\theta_{s,t}\widetilde{d}_t}{\eta_s}+\lambda_t(\eta_s^2\widehat{d}_t-\widetilde{d}_t)+\pi_t+M_s(\widetilde{d}_t-\eta_s^2\widehat{d}_t-\mu_t))$$ **Discharging**

Linear Relationship with Energy and Reserve Price

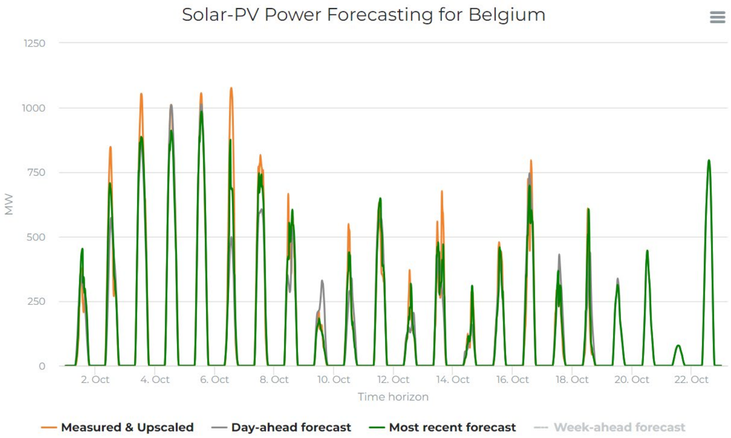
[1] N. Zheng, X. Qin, D. Wu, G. Murtaugh and B. Xu, "Energy Storage State-of-Charge Market Model," *IEEE Transactions on Energy Markets, Policy and Regulation*, vol. 1, no. 1, pp. 11-22, March 2023.
[2] Q., Xin, I. Lestas, and B. Xu. "Economic Capacity Withholding Bounds of Competitive Energy Storage Bidders." *arXiv preprint* arXiv:2403.05705 (2024).

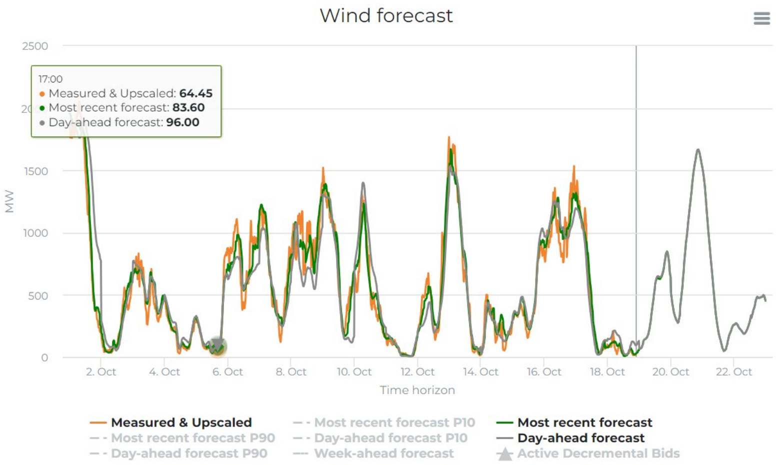
Case Study—Test System

- **ISO-NE 8-Zone Test System**



- **76 Generators: 23.1 GW**
- **Load: 13 GW**
- **Renewables: Wind and Solar[1], 10%-90% of Total Generation Capacity**
- **Uncertainty: Elia Historical Data[2]**
- **Multiple Storages: 10%-60% of Total Generation Capacity; 4-8-12 hr duration; 0.8, 0.85, 0.9, 0.95 One-Way Efficiency**
- **Coding[3]: MatLab and solved by Gurobi 11.0 solver, Intel Corei9-13900HX @ 2.30GHz with RAM 16 GB.**

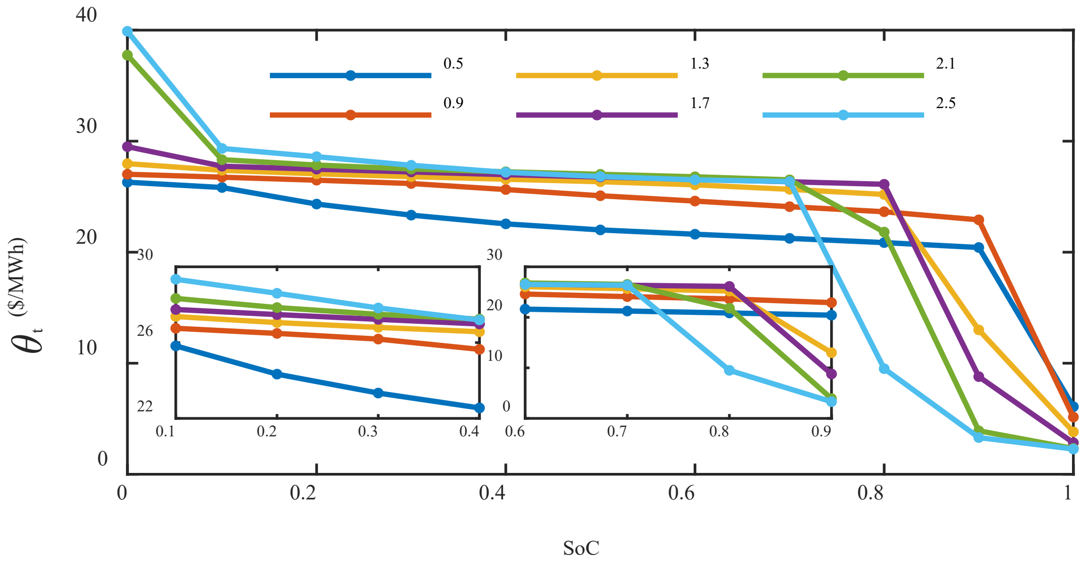




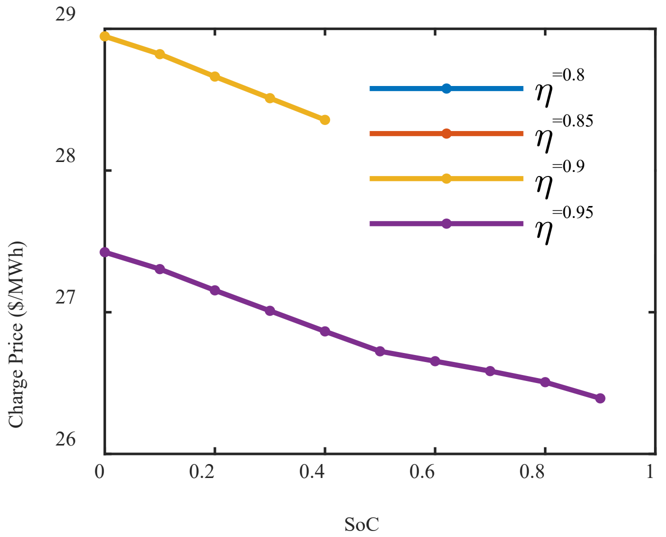
[1] D. Krishnamurthy, W. Li, and L. Tesfatsion, “An 8-zone test systembased on iso new england data: Development and application,” *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 234–246, 2015
[2] Elia, “Forecast error data from elia,” 2024. [Online]. Available:https://www.elia.be/en/grid-data.
[3] Code and Data: https://github.com/thuqining/Storage_Pricing_for_Social_Welfare_Maximization

Case Study—Simulation Results

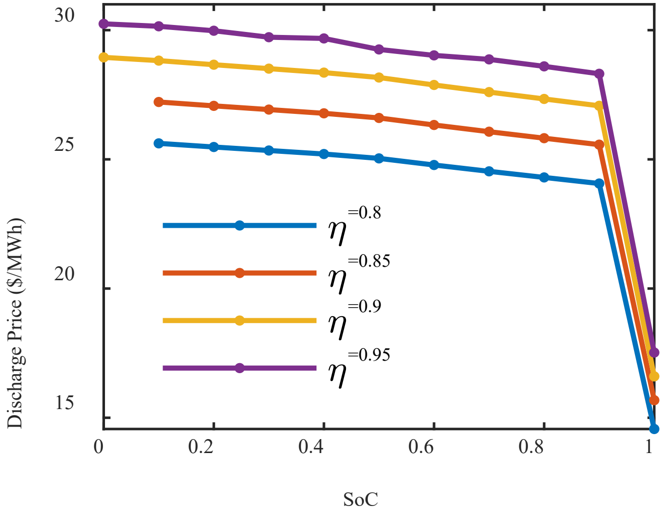
- **Convex Opportunity Price Function θ(SoC)**
- **Monotonically Decreasing**



- **Higher Efficiency: Higher Discharge Price, Lower Charge Price**



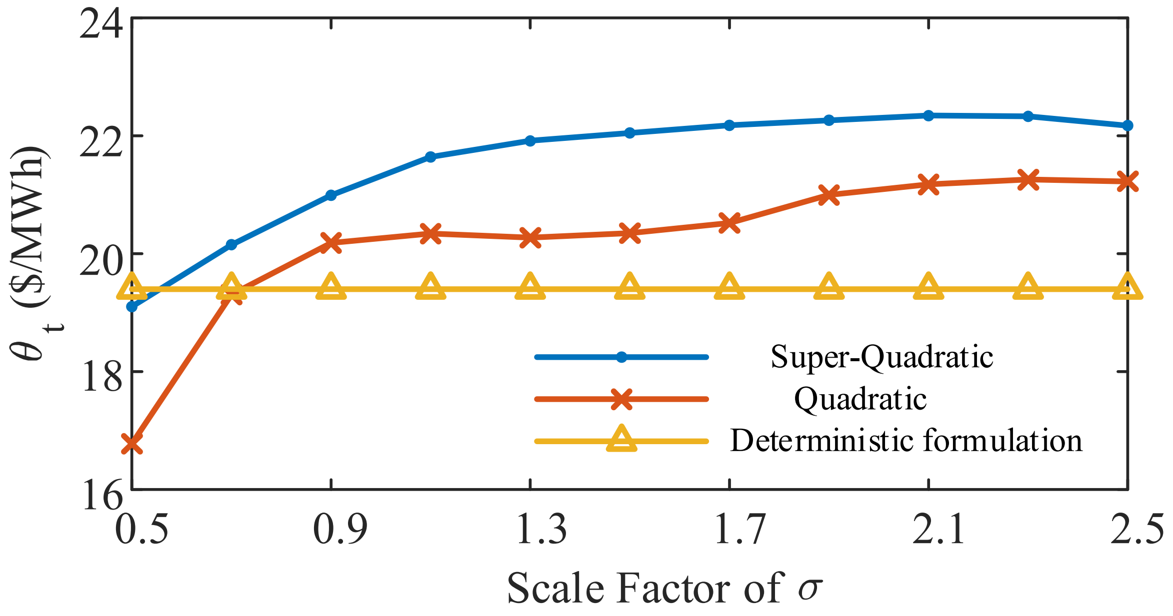
(a)



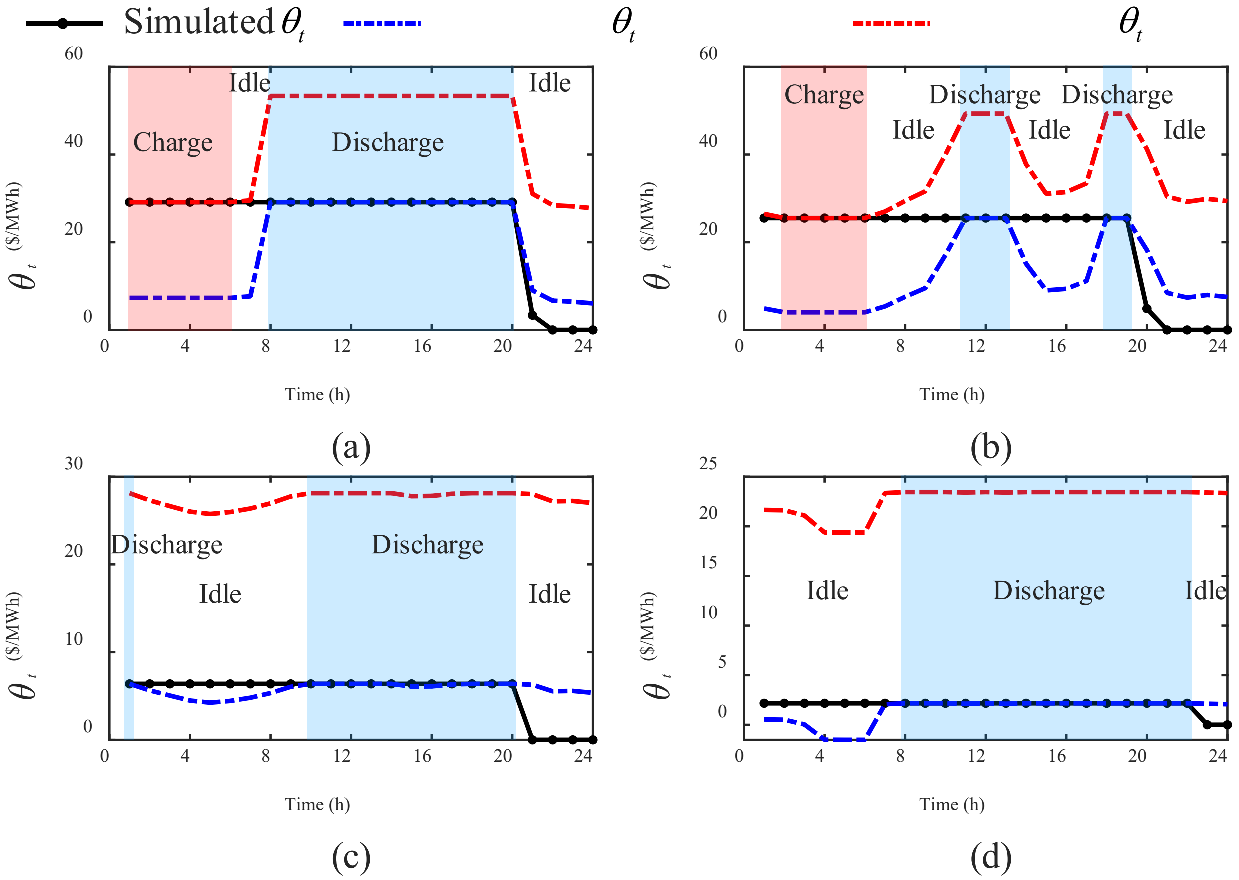
(b)

Case Study—Simulation Results

- **Uncertainty-Aware Price Function $\theta(\sigma)$**
- **Proposed Formulation: Monotonically Increasing**
- **Deterministic Formulation: Fixed**



- **Constrained and bounded Price**
- **Anticipated Default Bid**



Case Study—Comparative Results

- **Benchmark Comparison**
- **Storage Profit Maximization[1,2]**

$$V_{t-1}(e_{t-1}) = \max_{p_t,\, b_t} \lambda_t(p_t - b_t) - Mp_t + V_t(e_t)$$
$$e_{t+1} - e_t = -p_t/\eta + b_t\eta$$
$$0 \le b_t \le \overline{P},\ 0 \le p_t \le \overline{P}$$
$$0 \le e_t \le \overline{E}$$
$$p_t = 0,\ \lambda_t < 0$$

Opportunity Value Function

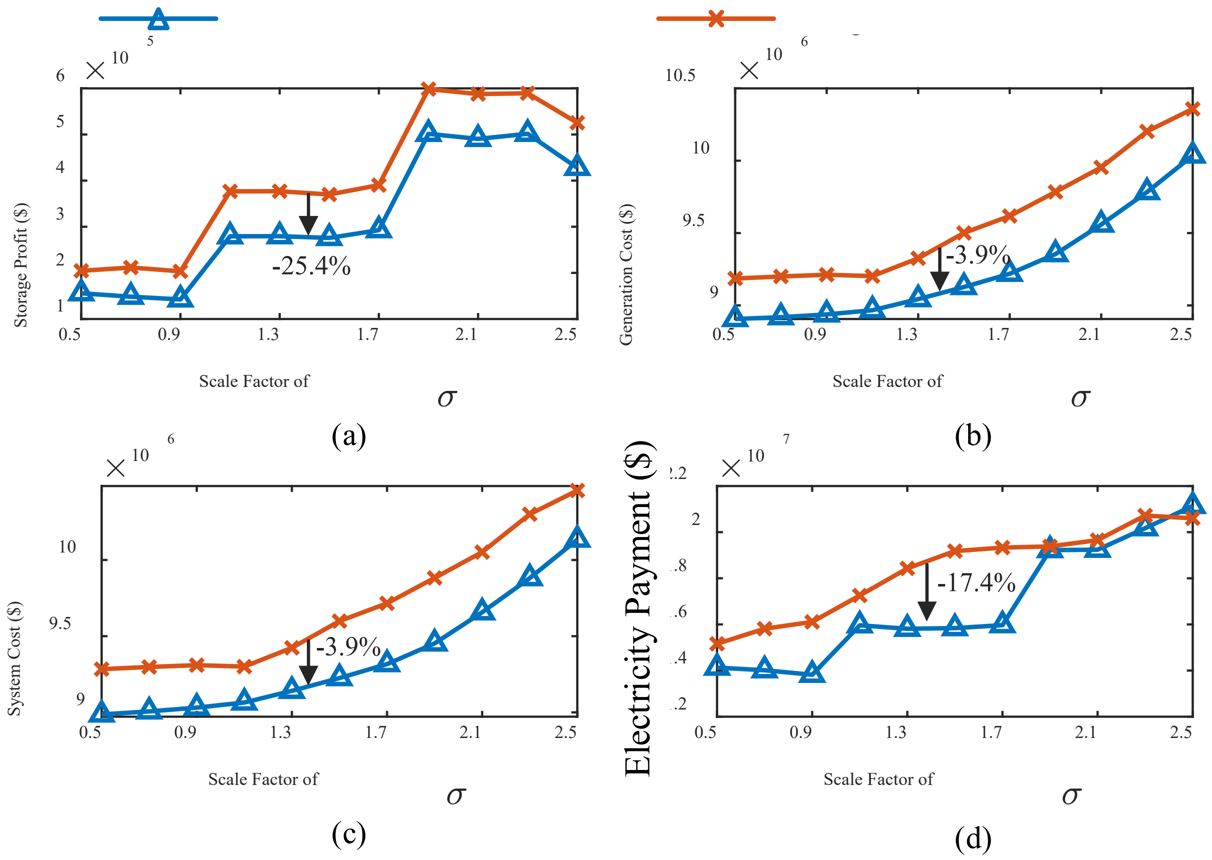
$$O_t(p_t) = \frac{\partial(Mp_t + V_t(e_t))}{\partial p_t} = M + \frac{1}{\eta} v_t(e_{t-1} - p_t/\eta)$$
$$B_t(b_t) = \frac{\partial(V_t(e_t))}{\partial b_t} = \eta v_t(e_{t-1} + b_t\eta)$$

Storage Bid

$$\min \sum_i \sum_t G(g_{i,t}) + O_t(p_t) - B_t(b_t)$$

Market Clearing

- **Enhance social welfare, reduce conventional generator production and consumer payment; Sacrifice storage margins**
- **Electricity Payment Decrease by 17%**
- **Storage Profit Reduces by 0.5% (based on Electricity Payment)**



[1] X. Qin, I. Lestas, and B. Xu, "Economic capacity withholding bounds of competitive energy storage bidders," arXiv preprint arXiv:2403.05705,2024

[2] N. Zheng, X. Qin, D. Wu, G. Murtaugh and B. Xu, "Energy Storage State-of-Charge Market Model," IEEE Transactions on Energy Markets, Policy and Regulation, vol. 1, no. 1, pp. 11-22, March 2023.

Case Study—Comparative Results

● **Benefit Scales Up with Increased Renewable & Storage Integration**

- **Sensitivity Analysis of Storage Capacity**

✓ **4-hr storage:**
20% storage capacity (-18%)

⇩

60% storage capacity (-27%)

✓ **20% storage capacity:**
4-hr storage (-18%)

⇩

12-hr storage (-23%)

- **Sensitivity Analysis of Renewable Capacity**

✓ **30% renewable capacity (-18%)**

⇩

✓ **50% renewable capacity (-21%)**

⇩

✓ **70% renewable capacity (-22%)**

Case Study—Comparative Results

- **Uncertainty Realization & Risk-Aversion**

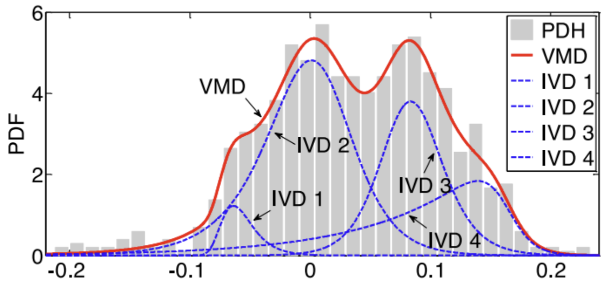
(1) Normal Distribution

$$F^{-1}(1-\epsilon) = \Phi^{-1}(1-\epsilon)$$

(2) Distributionally Robust

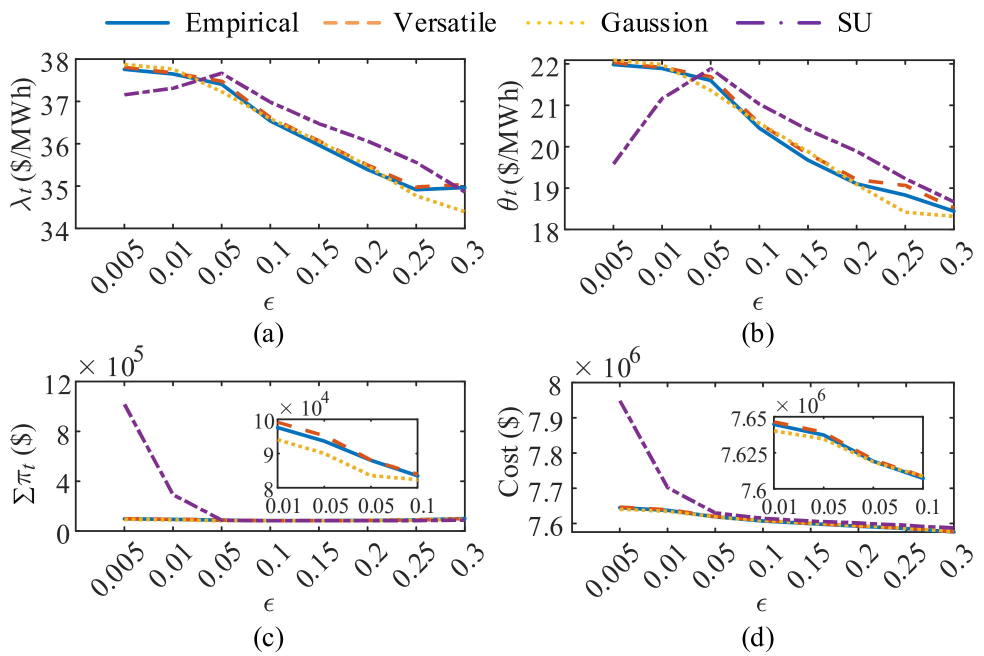
Type & Shape	$F^{-1}(1-\epsilon)$	ϵ
No Distribution Assumption (NA)	$\sqrt{(1-\epsilon)/\epsilon}$	$0<\epsilon\leq 1$
Symmetric Distribution (S)	$\sqrt{1/2\epsilon}$	$0<\epsilon\leq 1/2$
	0	$1/2<\epsilon\leq 1$
Unimodal Distribution (U)	$\sqrt{(4-9\epsilon)/9\epsilon}$	$0<\epsilon\leq 1/6$
	$\sqrt{(3-3\epsilon)/(1+3\epsilon)}$	$1/6<\epsilon\leq 1$

(3) Versatile Distribution[1]



[1] Z.-S. Zhang, Y.-Z. Sun, D. W. Gao et al., "A versatile probabilitydistribution model for wind power forecast errors and its applicationin economic dispatch," IEEE Transactions on Power Systems, vol. 28,no. 3, pp. 3114–3125, 2013.

- **More Risk-Aversion→Higher Cost & Prices**
- **Versatile Distribution Best Fits the Netload Forecast Error**



Distributions	Energy Price	Storage Price
Versatile	0.08	0.12
Gaussion	0.23	0.20
SU	0.48	1.00

Compared with Empirical Result Lower RMSE: Better Fitting

Case Study—Comparative Results

● Computational Efficiency

Generator Number	1	76	76	76	76	76	76	76	76	76
Storage Number	1	1	5	10	50	100	500	1000	5000	10000
Time1 (s)	0.08	0.42	0.57	0.78	5.63	11.86	370.00	424.45	>3000	>3000
Time2 (s)	0.04	0.35	0.46	0.59	0.74	1.16	2.85	5.32	33.34	72.60

Scalable!

Time1: Complementary Constraints to Prevent Storage from Simultaneous Charging and Discharging

Time2: Relaxation of Complementary Constraints[1]

$$\mathbf{E}^r(\mathbf{P}_c^r, \mathbf{P}_d^r) = \mathbf{1}_T E_0 + \eta_c \mathbf{A}\mathbf{P}_c^r - \frac{1}{\eta_d}\mathbf{A}\mathbf{P}_d^r$$

$$\mathbf{0} \leq \mathbf{1}_T E_0 + \eta_c \mathbf{A}\mathbf{P}_c - \frac{1}{\eta_d}\mathbf{A}\mathbf{P}_d$$

$$\mathbf{E}_{\max} \geq \mathbf{1}_T E_0 + \eta \mathbf{A}(\mathbf{P}_c - \mathbf{P}_d)$$

$$0 \leq \mathbf{P}_c \leq \mathbf{1}_T P_{\max}$$

$$0 \leq \mathbf{P}_d \leq \mathbf{1}_T P_{\max}$$

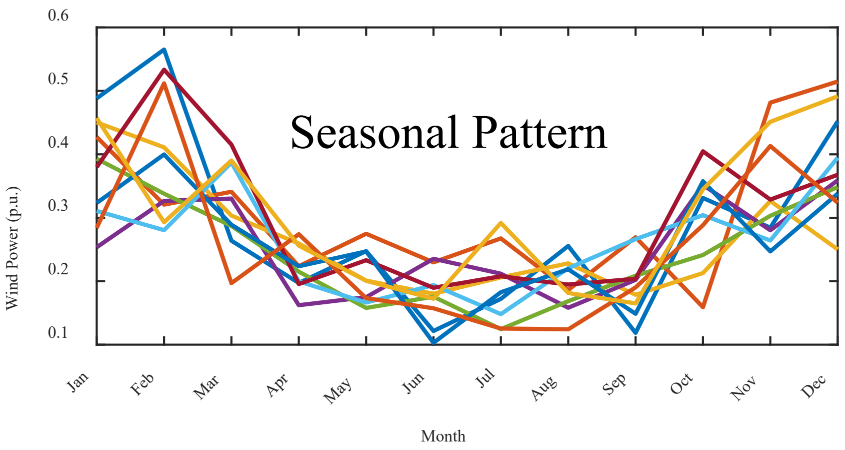
$$\mathbf{P}_c + \mathbf{P}_d \leq \mathbf{1}_T P_{\max}$$

[1] N. Nazir and M. Almassalkhi, "Guaranteeing a physically realizable battery dispatch without charge-discharge complementarity constraints," *IEEE Transactions on Smart Grid*, vol. 14, no. 3, pp. 2473–2476, 2021.

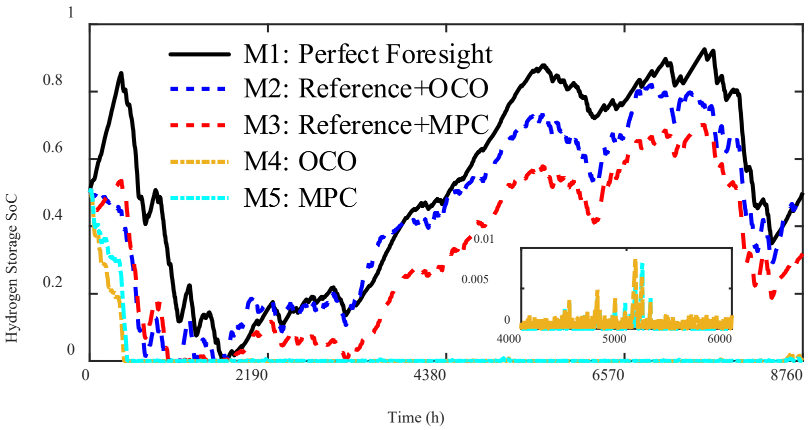
Conclusion & Future Work

- ✓ **Two-Stage Chance-Constrained Pricing Framework—Default Bid, Benchmark Market Power of Storage, Integrated into Current Economic Dispatch**
- ✓ **Theoretical Analysis: Convex, Uncertainty-Aware, Bounded and Anticipated**
- ✓ **Simulations: Significantly Enhance Social Welfare!—CASIO[1]**

? Long-Duration Storage—Track the Seasonal Pattern[3]



Annual Dispatch



[1] https://www.caiso.com/notices/new-initiative-storage-bid-cost-recovery-and-default-energy-bids-enhancements-workshop-call-on-7-8-24
[2] N. Qi, N. Zheng, and B. Xu. "Chance-constrained energy storage pricing for social welfare maximization." arXiv preprint arXiv:2407.07068.
[3] N. Qi, K. Huang, Z. Fan, and B. Xu. "Long-term energy management for microgrid with hybrid hydrogen-battery energy storage: A prediction-free coordinated optimization framework." Applied Energy 377 (2025): 124485.

Thank You!

Ning Qi

nq2176@columbia.edu

Columbia University, New York, USA

October 22, 2024

Theoretical Analysis—Deterministic Framework

Deterministic Formulation

$$\min \sum_t [\sum_i G_i(g_{i,t}) + \sum_s M_s(p_{s,t})] \quad (1a)$$

$$[\lambda_t]: \sum_i g_{i,t} + \sum_s p_{s,t} - \sum_s b_{s,t} = D_t,\ \forall t \in \mathcal{T} \quad (1b)$$

$$[\theta_{s,t}]: e_{s,t+1} - e_{s,t} = -p_{s,t}/\eta_s + b_{s,t}\eta_s,\ \forall t \in \mathcal{T},\ \forall s \in \mathcal{S} \quad (1c)$$

$$[\pi_t]: \sum_i \varphi_{i,t} + \sum_s \psi_{s,t} \geq d_t,\ \forall t \in \mathcal{T} \quad (1d)$$

$$[\underline{\nu}_{i,t}, \overline{\nu}_{i,t}]: \underline{G}_i \leq g_{i,t} \leq \overline{G}_i - \varphi_{i,t},\ \forall t \in \mathcal{T},\ \forall i \in \mathcal{G} \quad (1e)$$

$$[\underline{\kappa}_{i,t}, \overline{\kappa}_{i,t}]: -RD_i \leq g_{i,t} - g_{i,t-1} \leq RU_i,\ \forall t \in \mathcal{T},\ \forall i \in \mathcal{G} \quad (1f)$$

$$[\underline{\alpha}_{s,t}, \overline{\alpha}_{s,t}]: 0 \leq b_{s,t} \leq \overline{P}_s,\ \forall t \in \mathcal{T},\ \forall s \in \mathcal{S} \quad (1g)$$

$$[\underline{\beta}_{s,t}, \overline{\beta}_{s,t}]: 0 \leq p_{s,t} \leq \overline{P}_s - \psi_{s,t},\ \forall t \in \mathcal{T},\ \forall s \in \mathcal{S} \quad (1h)$$

$$[\underline{\iota}_{s,t}, \overline{\iota}_{s,t}]: (\psi_{s,t} + p_{s,t})/\eta_s + \underline{E}_s \leq e_{s,t}, \quad (1i)$$
$$e_{s,t} \leq \overline{E}_s - b_{s,t}\eta_s,\ \forall t \in \mathcal{T},\ \forall s \in \mathcal{S}$$

- **Proposition 1: Physical Cost Origins: (1) Marginal Cost of Cleared Generator & Storage (2) Storage Efficiency**

$$\theta_{s,t} = H_i(g_{i,t})/\eta_s$$ **Charging**

$$\theta_{s,t} = \eta_s(H_i(g_{i,t}) - M_s)$$ **Discharging**

- **Proposition 2: Convex Opportunity Price: Monotonically Decreases with Storage SoC**

$$\partial\theta_{s,t}/\partial e_{s,t} \leq 0$$ **Quadratic/Super-Quadratic Generation Cost Function**

- **Theorem: Constrained and Bounded Price**

$$\theta_{s,t} \geq \eta_s(\lambda_t - \pi_t - M_s),\ \theta_{s,t-1} = \lambda_t/\eta_s + \pi_t\eta_s$$ **Charging**

$$\theta_{s,t} = \eta_s(\lambda_t - \pi_t - M_s),\ \theta_{s,t-1} \leq \lambda_t/\eta_s + \pi_t\eta_s$$ **Discharging**

× **Uncertainty-aware Price**

Chance-constrained optimization

Deterministic Reformulation

$$\mathbb{P}\left(a_i(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\xi} \leq b_i(\boldsymbol{x})\right) \geq 1-\epsilon$$

$$a_i(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\mu} + b_i(\boldsymbol{x}) + \boxed{F^{-1}(1-\epsilon)}\sqrt{a_i(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\Sigma}a_i(\boldsymbol{x})} \leq 0$$

(1) Gaussion Distribution

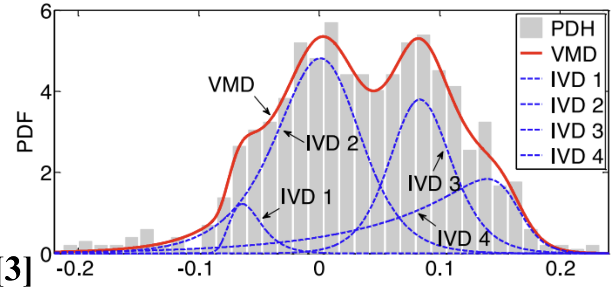
$$F^{-1}(1-\epsilon) = \Phi^{-1}(1-\epsilon)$$

(2) Distributionally Robust[1]

Type & Shape	$F^{-1}(1-\epsilon)$	ϵ
No Distribution Assumption (NA)	$\sqrt{(1-\epsilon)/\epsilon}$	$0<\epsilon\leq 1$
Symmetric Distribution (S)	$\sqrt{1/2\epsilon}$	$0<\epsilon\leq 1/2$
	0	$1/2<\epsilon\leq 1$
Unimodal Distribution (U)	$\sqrt{(4-9\epsilon)/9\epsilon}$	$0<\epsilon\leq 1/6$
	$\sqrt{(3-3\epsilon)/(1+3\epsilon)}$	$1/6<\epsilon\leq 1$
Symmetric & Unimodal Distribution (SU)	$\sqrt{2/9\epsilon}$	$0<\epsilon\leq 1/6$
	$\sqrt{3}(1-2\epsilon)$	$1/6<\epsilon\leq 1/2$
	0	$1/2<\epsilon\leq 1$

(3) Versatile Distribution[2]

$$F^{-1}(1-\epsilon \mid a,b,c) = c - \ln\left((1-\epsilon)^{-1/b} - 1\right)/a$$



(4) Data-Driven Distributionally Robust[3]

$$a_i(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\mu}(\boldsymbol{x}) + F_{\boldsymbol{x}}^{-1}(1-\epsilon)\sqrt{a_i(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\Sigma}(\boldsymbol{x})a_i(\boldsymbol{x})} \leq b_i(\boldsymbol{x})$$

$$a_i(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\mu}(\boldsymbol{x}) + \psi_K \boldsymbol{r}(\boldsymbol{x}) + \pi_K \overline{F}_{\boldsymbol{x}}^{-1}(1-\epsilon)\,\|\,\boldsymbol{y}\,\|_2 \leq b_i(\boldsymbol{x})$$

$$\sqrt{a_i(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\Sigma}a_i(\boldsymbol{x})} \leq y_1, \sqrt{2\psi_K}\boldsymbol{r}(\boldsymbol{x}) \leq y_2$$

$$\psi_K = K^{(1/p-1/2)}$$

$$\pi_K = \left(1 - \frac{4}{\epsilon}\exp\left(-\left(K^{1/p}-2\right)^2/2\right)\right)^{-1/2}$$

[1] N. Qi, P. Pinson, M. R. Almassalkhi, et al., "Chance-constrained generic energy storage operations under decision-dependent uncertainty," IEEE Transactions on Sustainable Energy, vol. 14, no. 4, pp. 2234–2248, 2023.

[2] Z.-S. Zhang, Y.-Z. Sun, D. W. Gao et al., "A versatile probability distribution model for wind power forecast errors and its applicationin economic dispatch," IEEE Trans. on Power Systems, vol. 28,no. 3, pp. 3114–3125, 2013.

[3] N. Qi, P. Pinson, M. R. Almassalkhi, et al., "Capacity Credit Evaluation of Generalized Energy Storage Considering Endogenous Uncertainty," IEEE Trans. on Power Systems (second review).